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Rigid and complete intersection Lagrangian singularities. (English summary)

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The authors develop their study of the local cohomology of the Lagrangian de Rham complex, introduced by themselves in [Math. Ann. **327** (2003), no. 1, 79–102; [MR2005122 \(2004k:32046\)](#)], to consider the deformation problem for a Lagrangian singularity. A rigidity theorem under Lagrangian deformations is obtained in dimension greater than two. It is also shown that for Lagrangian complete intersection singularities the Lagrangian de Rham complex is perverse.

As an application it is shown that all open swallowtails of dimension greater than one are rigid Lagrangian singularities. The open swallowtails were introduced by A. B. Givental [in *Current problems in mathematics. Newest results, Vol. 33 (Russian)*, 55–112, 236, Akad. Nauk SSSR, Vsesoyuz. Inst. Nauchn. i Tekhn. Inform., Moscow, 1988; [MR0967765 \(91g:58077\)](#)] as a special subvariety of a space of polynomials whose natural symplectic structure is related to the representation theory of $\mathfrak{sl}_2$.

Reviewed by [Alejandro Melle-Hernández](#)

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.